

## Mathematica 7 and Field Lines

Question 2009B.2 asked students to sketch the field lines of two identical positive charges  $+Q$  placed some distance apart on a line. It's not hard to visualize the picture and make the sketch, but for typed up solutions it would be nice to have a good looking diagram. *Mathematica* (v. 7) has the command `StreamPlot` to do this.

Let a positive charge  $Q$  be placed at the position  $(x_o, y_o)$ . The electric field  $\mathbf{E}$  at a point  $(x, y)$  is given by

$$\mathbf{E} = \frac{kQ}{r^2} (\cos \theta, \sin \theta)$$

where

$$r = \sqrt{(x - x_o)^2 + (y - y_o)^2} \quad \cos \theta = \frac{x - x_o}{r} \quad \sin \theta = \frac{y - y_o}{r}$$

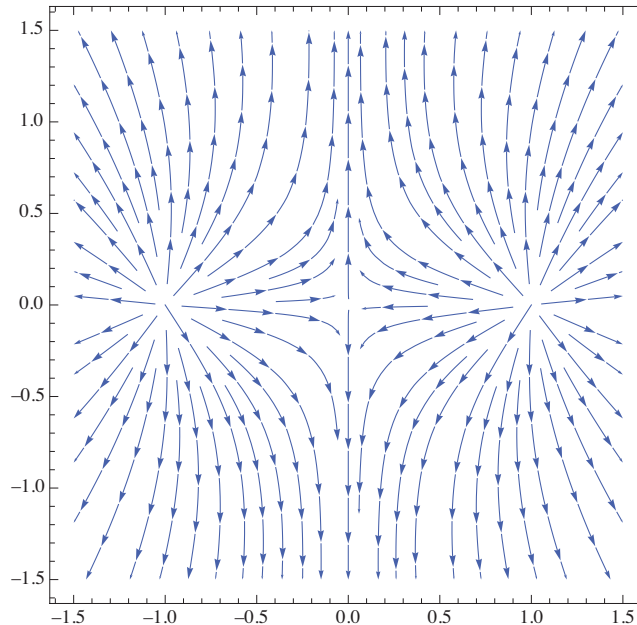
(and as usual,  $k = 1/4\pi\epsilon_o$ .) *Mathematica* needs to know where to draw the field lines. Put the charges on the  $x$ -axis, say at  $(x_1, y_1) = (-1, 0)$  and  $(x_2, y_2) = (1, 0)$ . The scale of the graph is immaterial, so we are asking the program to show us the field lines for the following: (with units such that  $kQ = 1$ )

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \left( \frac{x+1}{(\sqrt{(x+1)^2 + y^2})^3}, \frac{y}{(\sqrt{(x+1)^2 + y^2})^3} \right) + \left( \frac{x-1}{(\sqrt{(x-1)^2 + y^2})^3}, \frac{y}{(\sqrt{(x-1)^2 + y^2})^3} \right)$$

The command `StreamPlot[{Fx(x,y), Fy(x,y)}, {x, x1, x2}, {y, y1, y2}]` plots a field  $\mathbf{F} = (F_x(x, y), F_y(x, y))$  in the range  $x_1 \leq x \leq x_2$ ; entering

$$\text{StreamPlot}\left\{\left\{\frac{x-1}{\left(\sqrt{(x-1)^2 + y^2}\right)^3} + \frac{x+1}{\left(\sqrt{(x+1)^2 + y^2}\right)^3}, \frac{y}{\left(\sqrt{(x-1)^2 + y^2}\right)^3} + \frac{y}{\left(\sqrt{(x+1)^2 + y^2}\right)^3}\right\}, \{x, -1.5, 1.5\}, \{y, -1.5, 1.5\}\right\}$$

into *Mathematica* produces this:



It's not perfect, but it's pretty good. (Earlier versions of *Mathematica* did not have this command built in.)