(Handout 12a) The "Dumbot" Engine

One of the central ideas in thermodynamics is the celebrated result (1824) of the French engineer Sadi Carnot (pronounced "kar-NO"), who designed the world's most efficient engine. Carnot's ideas are difficult to follow. Here is a much simpler toy engine, which may be the world's worst, a "dumbo" engine, or in homage to the French genius, a "Dumbot" engine. If the calculations here make sense to you, then you will not find Carnot's engine and his ideas hard to grasp.

Imagine a quantity of an ideal monatomic gas like neon. We know for this gas $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$. We are going to put this gas in a cylinder with a piston, as usual. We will then take the gas through four processes. When we finish, the gas will back where we began; this is a *cyclic process*, or **cycle** for short.

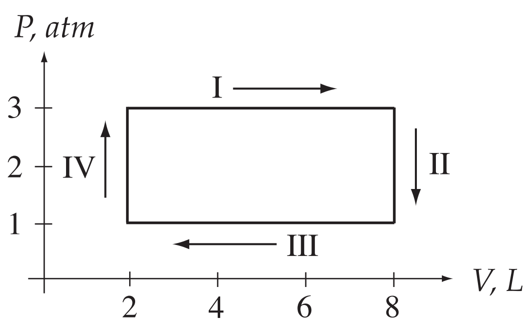
Step I. Isobaric expansion and heating from $(V_1, P_1) = (2\,\text{L}, 3\,\text{atm})$ to $(V_2, P_2) = (8\,\text{L}, 3\,\text{atm})$. Let's say that the original temperature T_1 of the gas is 600 K. Then using the Gay-Lussac Law, it follows quickly that $T_2 = 2400$ K. During this process, heat is added to the gas, and the gas expands, raising the piston and doing some work.

Step II. Isochoric cooling from $(V_2, P_2) = (8\,\text{L}, 3\,\text{atm})$ to $(V_3, P_3) = (8\,\text{L}, 1\,\text{atm})$. Find easily that $T_3 = 800$ K. During this process, heat is taken from the gas, by surrounding the gas with cold water or something to serve as a cold reservoir into which to dump heat. No work is done because the volume is held constant.

Step III. Isobaric compression and cooling from $(V_3, P_3) = (8\,\text{L}, 1\,\text{atm})$ to $(V_4, P_4) = (2\,\text{L}, 1\,\text{atm})$. Now $T_4 = 200$ K. Heat is again taken from the gas. Work is done on the gas as we or some other outside agency presses down on the gas. Notice that we do the compression after the gas is cooled, as it is easier to compress a cool gas than a hot one.

Step IV. Isochoric heating from $(V_4, P_4) = (2\,\text{L}, 1\,\text{atm})$ back to to $(V_1, P_1) = (2\,\text{L}, 3\,\text{atm})$. Now $T_4 = 200$ K. Heat is added to the gas, but no work is done.

These steps are most conveniently represented on a pressure–volume graph:



This particular cycle begins at the upper left hand corner of the rectangle, and moves clockwise. As will become evident, the clockwise path makes the cycle an **engine**. *An engine is a machine for turning heat into work.* Engines are based upon the trinity of heat, work and energy. We will go through each step of the cycle, and calculate ΔE, Q and W for each step, and for the cycle.

I. $\Delta E = nC_V\Delta T$. We need to calculate n. Note, however, that n rarely occurs except in products with the gas constant, R. That is, it is usually easier to calculate nR, rather than n. From the Ideal Gas Law,

$$PV = nRT \to nR = \frac{PV}{T} = \frac{(3\,\text{atm} \times 2\,\text{L})}{600K} = \tfrac{1}{100}\,\text{atm-L/K}$$

Using $C_V = \frac{3}{2}R$ given above, we find

$$\Delta E = nC_V\Delta T = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}\tfrac{1}{100}\,\text{atm-L/K} \times (2400 - 600)\text{ K} = 27\,\text{atm-L}$$

(Reminder: Since 1 atm = 1.013×10^5 Pa, and 1 L = 10^3 m^3, 1 atm-L = 101.3 J.) For constant volume processes,

$$W = -P\Delta V = -(3\,\text{atm}) \times (8\,\text{L} - 2\,\text{L}) = -18\,\text{atm-L}$$
$$Q = nC_P\Delta T = \tfrac{5}{2}nR\Delta T = \tfrac{5}{2}\tfrac{1}{100}\,\text{atm-L/K} \times (2400 - 600)\,\text{K}) = 45\,\text{atm-L}.$$

Recall that with our conventions, a *negative* quantity of energy means that the gas loses it; a *positive* quantity means that the gas gains it. If the net work is negative, as it is here, then *the gas does the work.* Note also that at every step, we have to satisfy the First Law of Thermodynamics, $\Delta E = Q + W$, and we do here: 27 atm-L = 45 atm-L −18 atm-L.

II. $\Delta E = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}\frac{1}{100}\,\text{atm-L/K} \times (800 - 2400)\,\text{K}) = -24\,\text{atm-L}$. For constant volume processes,

$$W = 0\,\text{atm-L (since } \Delta V = 0)$$
$$Q = \Delta E = nC_V\Delta T = -24\,\text{atm-L}.$$

III. $\Delta E = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}\frac{1}{100}\,\text{atm-L/K} \times (200 - 800)\,\text{K}) = -9\,\text{atm-L}$. As in I., we have a constant pressure process;

$$W = -P\Delta V = -(1\,\text{atm}) \times (2\,\text{L} - 8\,\text{L}) = +6\,\text{atm-L}$$
$$Q = nC_P\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}\frac{1}{100}\,\text{atm-L/K} \times (200 - 800)\,\text{K}) = -15\,\text{atm-L}$$

Does this satisfy the First Law? Yes: $-9\,\text{atm-L} = (-15 + 6)$ atm-L.

IV. $\Delta E = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}\frac{1}{100}\,\text{atm-L/K} \times (600 - 200)\,\text{K}) = 6\,\text{atm-L} = Q$, because no work is done.

Let's put all this into a table (all in units of atm-L):

process	ΔE	Q	W
I.	27	45	−18
II.	−24	−24	0
III.	−9	−15	6
IV.	6	6	0
net	0	12	−12

What this engine did for us is to convert a *net* of 12 atm-L of heat (obtained by burning coal, wood or whatever) to 12 atm-L of useful work. Overall there is no loss of energy; our cycle (and *any* cycle) has a net $\Delta E = 0$. What is the *efficiency* of this engine?

Newcomers to thermodynamics look at the chart, and say: the efficiency is 100%, because a cost of 12 atm-L in heat produced a net of 12 atm-L of work. Absolutely not! What did it cost us to run this engine? In Step I, we burned up 45 atm-L, and in Step IV, another 6 atm-L. That means we invested 51 atm-L of heat into this engine. What about the −24 atm-L and the −15 atm-L in Steps II and III? That heat was dumped into the cold reservoir *as exhaust*; it is gone forever and cannot be regained without spending even more than was lost. The profit for this engine, a measly 12 atm-L, was obtained from an investment not of 12 atm-L, but instead of 51 atm-L. That is, in this case

$$\text{efficiency} = \frac{\text{(profit)}}{\text{(investment)}} = \frac{12\,\text{atm-L}}{51\,\text{atm-L}} = 23.5\%$$

The rule for an engine's efficiency is easy to state:

$$\boxed{\text{efficiency} = \frac{|W_{\text{net}}|}{Q_{\text{hot}}}}$$

where Q_{hot} = the sum of all the *positive* Q's. We need the absolute value brackets on W_{net} because for an engine this is always negative. This definition of efficiency is very reasonable, especially if put in monetary terms. A person who earns \$20 on an investment does better than one who earns \$30 on \$200. The second person made more money, but the first person is the better investor (20% vs 15%.)

Note that if the engine were run in reverse, the net work would have been +12 atm-L, and the net heat would have been −12 atm-L. That means that we would have done 12 atm-L on the gas, and for our trouble, 12 atm-L would have been *extracted* from the cold reservoir: this is a *refrigerator*, and the cycle would have gone *counter-clockwise* on our PV diagram.

This engine is not very efficient. Carnot showed how to make a better one, and in the process, discovered an astounding fact: *there is a limit to the efficiency of an engine*, as we will see.