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## Radioactive Half-Life

**Purpose**— To gain insight for the meaning of the terms *half life* ( $t_{1/2}$ ), *decay constant* ( $\lambda$ ), and *activity* ( $dN/dt$ ), using non-radioactive tools of probability (dice) instead of actual radioisotopes.

**Introduction**— What does rolling dice have to do with radioactivity? It turns out that the *rate* at which radioactive nuclei disintegrate has been found to be an immutable property of each isotope and is characteristic of that isotope. The decay process is statistical in nature, *i.e.*, for any single atom of the sample, there is a definite *probability* that it will disintegrate within a certain time interval, but one cannot actually predict or associate a precise instant in time for a given disintegration. Since the probability doesn't seem to depend on either the past history or the environment of the nucleus in question, statistical laws apply (*just as statistical laws apply to rolling dice*). Given the large number of nuclei comprising the sample or radioisotope, there is a definite probability that some particular proportion of disintegrations will occur in a given time interval. Hence the rate of disintegration is proportional to the number of radioactive nuclei initially present. The decay rate can be expressed mathematically as

$$\frac{\Delta N}{\Delta t} = \lambda N \quad (1)$$

where  $\frac{\Delta N}{\Delta t}$  is the decay rate, also called the *activity* of the isotope, N is the number of radioactive

nuclei present, t is the time, and  $\lambda$  is the proportionality constant, called the *decay constant*, which is characteristic of the particular isotope. *The decay constant  $\lambda$  is the probability that a single nucleus will decay in a given period of time (usually seconds)*. (These explanations of what the decay constant, the activity, and the number of radioactive nuclei present would make excellent questions on a pop quiz.) In words, *how many decays* occur depends on two things—the *probability* that any one atom will decay and the number of atoms that could decay that are present in the sample to start with.

Since we're talking about randomness and probability, let's couch the discussion in terms of what happens with rolling dice. The *probability* that a fair die (meaning one that has not been tampered with and which is therefore truly random) will roll a specific number when thrown depends on the number of sides the die has. For example, if the number 3 is wanted to appear on the die, a 6 sided die has a 1 in 6 probability of rolling a 3 whereas a 20 sided die has a 1 in 20 chance for rolling a 3. The larger the number of sides that the die has, the less likely that the die will roll the specified number. (It is less likely for a 3 to appear on the face of a 20 sided die than on the face of a 4 sided die.)

Here's a specific example. Think of a normal 6 sided dice. Let's say that you take 100 of these six-sided dice and rolled them all at once. Then you remove all the dice that rolled a "3". How many dice of the original 100 would you expect to remove? Since the probability that a 3 would be rolled on a six sided dice is 1 in 6 (or 1/6), the number you'd expect to pull out is 1/6 of 100 dice, or about 16 or 17 (your calculator gives  $100 \div 6 = 16.\bar{6}$ )

Repeating all this in terms of Equation (1), out of 100 dice, you'd expect 1/6 of them to roll a three. Out of 100 dice, you'd expect to pull out 16 or 17 dice. In this cases,  $\frac{\Delta N}{\Delta t} = 17$ ,  $N = 100$ , and  $\lambda = \frac{1}{6}$ . If you substitute these values into Equation (1) above, here's what you get:

$$17 \approx \frac{1}{6} \times 100$$

because you removed 1/6 of the original 100 dice you started with, about 17 dice.

Now, let's tie this back into our discussion about radioactivity. In this example, the 100 dice stand for the number of atoms you started out with originally. The 17 dice that rolled a 3 represent the number of atoms that underwent radioactive decay during the first time period of observation. We would say that the *activity* of this sample was 17 decays per roll, because that is the number of dice that actually rolled a 3 out of the original sample.

Now Equation (1) will be manipulated to obtain some practically useful results that can be measured in the laboratory. You will not be asked to reproduce what follows, but it is instructive to see where the equations come from.

In the limit of shrinking time intervals (this should have some meaning for you folks taking calculus) Equation (1) becomes

$$-\frac{dN}{dt} = \lambda N$$

(a negative sign has been inserted to account for decreasing numbers of nuclei). If the equation is rearranged to get the 'N' terms together, one obtains

$$\frac{dN}{N} = -\lambda dt \quad (2)$$

Equation (2) may be integrated using calculus to obtain (after the smoke clears) *the decay equation*

$$N(t) = N_0 e^{-\lambda t} \quad (3)$$

Given the initial number of unstable nuclei,  $N_0$ , present at time  $t=0$  (when the experiment began), and knowing the decay constant  $\lambda$ , this equation allows the prediction of the number of unstable nuclei still present at some future time  $t$ . (As an aside, in actual fact the experimental determination of half life involves the *activity* of the sample, *i.e.*, the number of disintegrations per time interval that a detector actually counts. It turns out that the form of the activity equation is similar to Equation 3 given above.)

*The half life of a sample is defined as the amount of time that passes from the beginning of an experiment in which there is an initial number of atoms,  $N_0$ , to when half that number remain in the sample, *i.e.*, when  $N = \frac{1}{2} N_0$ . (This description of half life would also make a good question on a pop quiz.)*

Therefore, to determine the half-life of a radioisotope, all that must be known is the decay constant. Half lives of real samples vary from fractions of a second to millions of years (another possible pop-quiz question), and there is no real way to predict what isotope will have a long or a short half life. Experimental measurements provide the numerical values of half lives used by nuclear scientists.

The *half life* and the *activity* of the sample are related to each other mathematically. The relationship may be obtained by substituting  $N = \frac{1}{2} N_0$  and  $t = t_{1/2}$  into Equation 3 above ( $t_{1/2}$  represents the half life):

$$\begin{aligned} \frac{1}{2} N_0 &= N_0 e^{-\lambda t_{1/2}} \\ \frac{1}{2} &= e^{-\lambda t_{1/2}} \quad (4) \end{aligned}$$

Taking the natural logarithm of both sides of Equation 4, one gets (recall that  $\ln(e^z) = z$ ):

$$\begin{aligned} \ln\left(\frac{1}{2}\right) &= \ln\left(e^{-\lambda t_{1/2}}\right) \\ -0.6931 &= -\lambda t_{1/2} \end{aligned}$$

Finally, solving for the half life, one obtains the relationship between half life and decay constant,

$$t_{1/2} = \frac{0.6931}{\lambda} \quad (5)$$

In this experiment, each die is intended to represent an atom of a particular radioactive isotope. You will take 100 atoms of the same "isotope" and determine the "half life" and "activity" of this isotope. The

half life you find will be measured in terms of a number of rolls (instead of in terms of time as is done with real radioisotopes). The activity will be measured in the number of “disintegrations per roll” (instead of in “disintegrations per second” in the case of real radioisotopes).

For practical purposes, the most important equations used at this level of study are Equations (1), (3), and (5). The other ones are given as intermediate steps in the derivation process.

### Procedure

Equipment needed—100 dice (either 20, 12, 10, 8, 6, or 4 sided). Note: singular—*die*, plural—*dice*

- 1) Take 100 dice of the same number of sides (you may select what kinds of dice to use). Work at a space where you can spread out all 100 dice at once. Count your dice to make sure that you start with 100. Inform your instructor if you have a different number than 100 dice to start with.
- 2) Make a table in your lab book that records the number of dice you have, the number of dice you have removed (the *activity*), and the number of rolls you have done. The first entry in this table is that after 0 (zero) rolls, there are 100 dice remaining and 0 (zero) dice removed. (The following table for a series of rolls using 12 sided dice is an example. The first row has been completed for you.)

#### 12 Sided Dice

Roll Number	Activity (Number of Dice Removed)	N (Number of dice remaining at the end of the roll)
0	0	100
1		
2		

- 3) Roll all the dice for the first time (Roll #1). Remove all the dice in which the number “3” is showing (or you can choose which number *you* want to remove, but it must be the *same* number throughout the experiment with this type of dice). (For the purposes of this experiment, we are assuming that a given atom (die) decays or disintegrates when the specified number is rolled.) In your lab notebook record the number of dice you removed (the *activity*). Also record the number of dice remaining at the end of this roll. Set aside the removed (disintegrated) dice, and do not add them back into the dice that you roll again.
- 4) Roll the remaining dice again. Again remove all the “3s”, and record the number you removed on this roll. Again record the number of dice remaining, and pull the “decayed” dice from the pile and set them aside.
- 5) Repeat the rolling procedure, removing all the “3s” and recording the number you removed from the pile as well as the number remaining. Continue this procedure until you have as few dice remaining as there are sides to the dice you are rolling. (So, for example, if you are rolling 12 sided dice, you may stop when there are 12 or fewer dice remaining.)
- 6) Repeat Steps 1-5 with two other kinds of dice. When you finish, you should have 3 sets of data.
- 7) Use graphing software (such as Microsoft Excel) to graph the number of dice remaining (vertical axis) as a function of the roll number (horizontal axis). Your first value for all three curves should be (Roll #0, 100 Dice Remaining). Put all three curves on the same graph, and get the software to fit an exponential curve/trendline to each set of your data. Display the trendline equations on your graph.

- 8) Determine the *decay constant*  $\lambda$  from the best-fit equations you obtained. (Hint—Use Equation 3 to guide you. Your best-fit equation has this form.) You will have 3 values for decay constants, one for each type of dice you rolled.
- 9) Calculate the “half life” of each kind of dice using your best-fit trendline-determined decay constant  $\lambda$  (Hint—Use Equation 5 relating half life to the decay constant derived earlier in this description.)

**Questions**—Answer these in your lab notebook.

- 1) This is a lab that simulates radioactive decay. a) What do the dice represent in this experiment?  
b) What does a die that rolls a ‘3’ represent?
- 2) a) In each of your 3 die types, how many *actual* rolls did it take before 50 of the 100 original dice had been removed? (Hint—You must use your graph to help you. Draw a horizontal line at 50 Remaining Dice on the vertical axis to your curve. From where this line hits your curve, sketch a vertical line down to the Roll Number horizontal axis. The number where the vertical line hits this axis is the ‘half-life’ of your experiment. It will likely be a non-integer.) b) Compare the number of *actual rolls* that comes from the graph to the *calculated “half life”* you figured out earlier in Step 9 in the lab by calculating the % difference. c) Comment on how well these two numbers match for each of your 3 die types. (You will have 3 calculations.)

$$\% \text{ difference} = \frac{|\text{measurement}_1 - \text{measurement}_2|}{(\text{average of measurement}_1 \text{ and measurement}_2)} \times 100$$

- 3) a) What is the numerical value for the *ideal decay* constant for each of the dice you used? (Hint—It is related to the number of sides your dice had.) b) Calculate the % error between the decay constant you found in Step 8 for each kind of die and the *ideal* (theoretical) decay constant for that type of die. (You will have 3 calculations.)

$$\% \text{ error} = \frac{|\text{theoretical} - \text{experimental}|}{\text{theoretical}} \times 100$$

- 4) How does the half life of a die with a large number of sides compare with the half life of a die with a smaller number of sides? Explain why this is the case.

**Summary**—Write a closing statement to wrap up this lab. In your paragraph, explain what the different types of dice represent. Explain what is represented when a ‘3’ is rolled. Also restate the types of dice you used, and give the half life and decay constant you found for each die type. Comment on the agreement of your data with what you’d expect ideally.